7.3

A Rational Approach

Exploring Rational Functions Graphically

LEARNING GOALS

In this lesson, you will:

- Graph rational functions.
- Determine graphical behavior of rational functions from the form of the equation.
- Translate rational functions.

The word "rational" means to be sensible or reasonable. Humans are said to be rational beings for our ability to use logic to move from a problem to its solution. Most definitions for the word "rational" are subjective. What might seem rational to one person may seem completely irrational to another. A person or group may come up with an idea that seems perfectly reasonable to them, but may seem eccentric to another group. This sometimes makes coming to a decision, or devising a solution path, very difficult for large groups of people.

Have you ever come up with an idea that you thought was perfectly rational, but others didn't quite agree with your rationale? Were you able to use logic to convince them that your idea made sense?

PROBLEM 1 Slow Down, Asymptotic Curves Ahead!

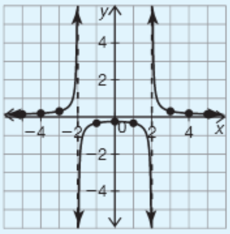
1. Analyze the methods Jodi, Theresa, and Jin each used to graph the rational function $j(x) = \frac{1}{x^2 - 4}$.

Jodi

I created a table and plotted the points.

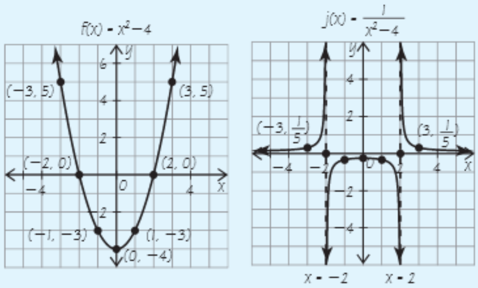
−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6
$\frac{1}{32}$	$\frac{1}{21}$	$\frac{1}{12}$	$\frac{1}{5}$	undefined	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{3}$	undefined	$\frac{1}{5}$	$\frac{1}{12}$	$\frac{1}{21}$	$\frac{1}{32}$

I see that vertical asymptotes occur at $x = -2$ and $x = 2$, where the denominator is 0 and the output is undefined.



Theresa

I graphed the function $f(x) = x^2 - 4$. The function $j(x)$ is the reciprocal of $f(x)$, so I took the reciprocal of several key points and sketched the graph.

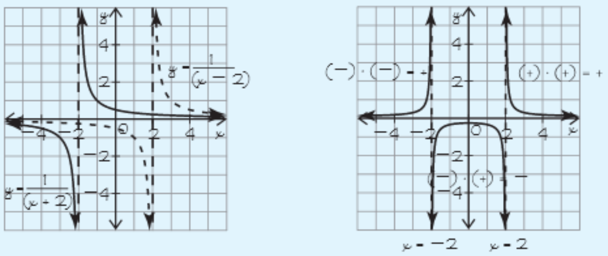


The zeros of the function $f(x) = x^2 - 4$ are $(-2, 0)$ and $(2, 0)$ that become asymptotes at $x = -2$ and $x = 2$ in the function $j(x) = \frac{1}{x^2 - 4}$. The y-intercept shifts from $(0, -4)$ in $f(x)$ to $0, \frac{-1}{4}$ in $j(x)$. By plotting a couple key points and recognizing that a horizontal asymptote is at $y = 0$, I can sketch the function.

Note that these aren't really "new" methods. Each student is applying previous knowledge to a new situation.

Jin

I used what I know about rational functions and function-building. Since the function $f(x) = \frac{1}{x^2 - 4}$ can be rewritten as two separate factors, $f(x) = \left(\frac{1}{x+2}\right)\left(\frac{1}{x-2}\right)$, I graphed each factor separately and multiplied their outputs to determine the graph of their product.



The asymptotes are at $x = -2$ and $x = 2$. Analyzing each function, I saw that the outputs were both negative for the interval $(-\infty, 2)$. Their product will always be positive so $f(x)$ will be above the x-axis for this region. Similarly, a positive and a negative output for the interval $(-2, 2)$ will always be negative. Two positive outputs multiplied together will be positive for the interval $(2, \infty)$.

a. Which method do you think is most efficient? Explain your reasoning.

b. Which method do you think is the most accurate? Explain your reasoning.

c. How does a vertical asymptote affect the domain of a function?

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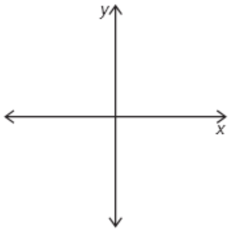
2. Without using a graphing calculator, sketch each function. Indicate the domain, range, vertical and horizontal asymptotes, and the y-intercept for each function.

a. $f(x) = \frac{1}{(x - 2)(x + 4)}$



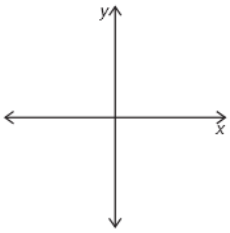
Domain:

Range:

Asymptote(s):

y-intercept:

b. $f(x) = \frac{2}{x^2 - 2x - 8}$



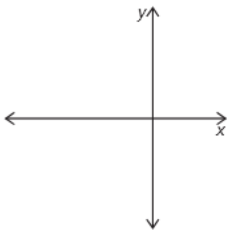
Domain:

Range:

Asymptote(s):

y-intercept:

c. $h(x) = \frac{1}{x^2 + 3x - 10}$



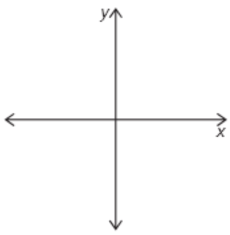
Domain:

Range:

Asymptote(s):

y-intercept:

d. $h(x) = \frac{1}{x^3 - 1}$



Domain:

Range:

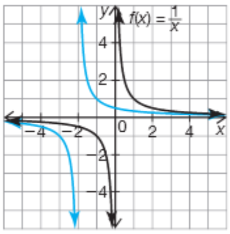
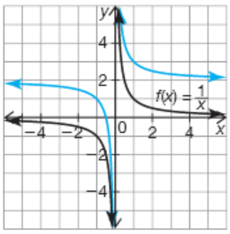
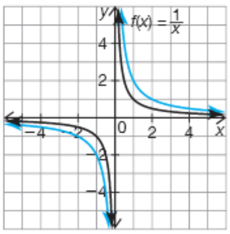
Asymptote(s):

y-intercept:

PROBLEM 2 Ctrl-Alt-Shift

Consider the functions $y = f(x)$ and $g(x) = Af(B(x - C)) + D$. Recall that adding a constant D translates $f(x)$ vertically, while adding a constant C translates $f(x)$ horizontally. Multiplying by the constant A dilates $f(x)$ vertically, while multiplying by the constant B dilates $f(x)$ horizontally. Rational functions are transformed in the same manner.

1. The function $f(x) = \frac{1}{x}$ is shown in black on each coordinate plane. Determine whether the second function on each graph is $j(x) = \frac{1}{x + 2}$, $m(x) = \frac{2}{x}$, or $k(x) = \frac{1}{x} + 2$. Explain your reasoning.

	Function: Explanation:
	Function: Explanation:
	Function: Explanation:

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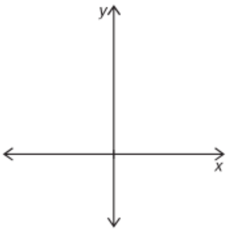
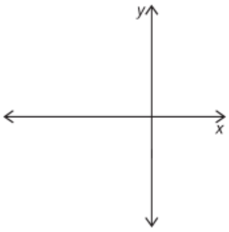
2. Given $f(x) = \frac{1}{x}$.

a. Sketch $g(x) = f(x) + 5$



Explanation:

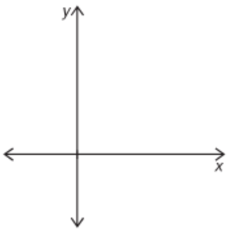
b. Sketch $h(x) = f(x + 5)$.



Explanation:

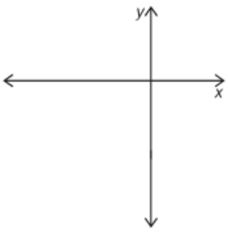
3. Write a rational function $g(x)$ that matches the given characteristics. Sketch the function on the coordinate plane.

a. Vertical asymptote at $x = 2$
Horizontal asymptote at $y = 1$



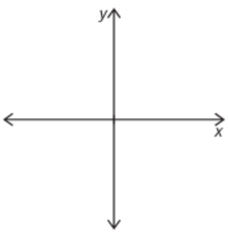
$g(x) =$

b. Vertical asymptote at $x = 1$, $x = -5$
Horizontal asymptote at $y = -3$



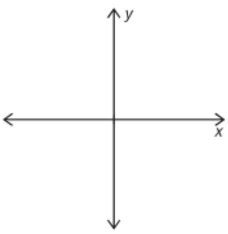
$g(x) =$

c. For $f(x) = \frac{1}{x}$, $g(x) = f(x - 2) - 4$



$g(x) =$

d. For $f(x) = \frac{1}{x}$, $g(x)$ shifts $f(x)$ up and to the left.



$g(x) =$

Be prepared to share your solutions and methods.